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Syllabus
Math 492
History of Mathematics

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Meeting Time: T 6:00 – 9:00 p.m.
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Course Description:

Math 492, History of Mathematics is a required Foundational Studies Upper Division Integrative Elective for Mathematics Education majors and is a Foundational Studies Upper Division Integrative Elective for any major (as long as the mathematics prerequisite and the class status prerequisite is met). In this course, you will have the opportunity to pull together many of the seemingly disparate "ways of knowing" components of your foundational studies experience including history, social and behavioral sciences (mainly sociology and anthropology), the natural sciences (mainly astronomy and physics), the arts and humanities (mainly philosophy, music, and art), and mathematics.

As an upper division integrative elective in the Foundational Studies Program, with a theme of history of mathematics, we will be studying mathematics. Hence, we will examine not only the people and the contexts in which they created mathematics, but we will also examine the actual mathematics itself: what it was, how it was developed, how it was conceptualized. You will have numerous opportunities to do independent research and collaborate with me on presentations, homework, and grading. As prospective teachers of secondary mathematics, it is important that you have an understanding of the history and development of your subject. This will enable you to put the mathematics that you teach and learn in an historical and "real world" context. Viewing mathematics from this perspective will help you understand mathematics as a human construct, created by people to help them make sense of and change their world.

By the end of Math 492 you should have a basic background in the history of mathematics. You will have learned some new mathematics and you will have gained new insights on some of the mathematics you had learned prior to this course. You will have gained some experience making substantial oral and written presentations. You will have become aware of how you can use history to help secondary students better appreciate and understand the mathematics you will be teaching them. You will have applied information within and across multiple "ways of knowing". Finally, you will be able to make connections between mathematics and other subjects and you will understand and appreciate the individual and social processes of creating new mathematics and how this impacts our daily lives.

Upon completion of the Foundational Studies program, you will have met all of the Foundational Studies Learning Objectives (detailed on the foundational studies website: http://www.indstate.edu/gened). The Foundational Studies program is designed to prepare you for the challenges you will face in the workplace and as an informed citizen.

In our work in Math 492, we will address the following Foundational Studies learning objectives to varying degrees:

- Locate, critically read, and evaluate information to solve problems.
- Critically evaluate the ideas of others
- Apply knowledge and skills within and across the fundamental ways of knowing (natural sciences, social and behavioral sciences, arts and humanities, mathematics and history)
- Demonstrate an appreciation of human expression through literature and fine and performing arts.
- Demonstrate an understanding of diverse cultures within and across societies.
- Demonstrate the skills to place your current and local experience in a global, cultural, and historical context.
- Demonstrate an understanding of the ethical implications of decisions and actions.
- Express yourself effectively, professionally, and persuasively both orally and in writing.

In addition, we will address the following learning objectives that specifically relate to Upper Division Integrative Electives:

- Use a thematic approach to a particular topic or issue that integrates multiple ways of knowing.
- Engage in a project or conduct research that makes use of multiple ways of knowing to address a particular topic or issue.
- Analyze and write at an advanced level.

We will also address each of the following skill and applied learning requirements:

- develop critical thinking skills
- develop information literacy skills
- include a developmental, graded writing component
- incorporate opportunities for you to critically read and analyze sophisticated, complex text, and to write intensively
- include assignments that apply information from within and across various "ways of knowing"

Course Requirements and Grading:

Attendance and Participation: Our class will draw on readings, problem explorations, collaborative and independent work both in and out of class. Your participation in class activities and discussions is important not only for your own learning, but also for the learning of others. Experiences during the class cannot be duplicated by reading a textbook. You are expected to treat our class as part of your professional experience; that is, you are expected to take responsibility for your learning and act in a professional and collegial manner. This includes attending each class on time with readings and

homework completed, and being actively involved in class. When you are using your laptop during class, you will only use if for course-related activities. If I find that you are using your laptop inappropriately, (i.e. checking email, facebook, etc.) I will ask you to leave the class for the evening. Please turn off cell phones during class out of respect for me and your classmates. If you have a situation that might require you to have your cell phone on, please notify me ahead of time. I reserve the right to lower your grade if you do not meet the aforementioned guidelines.

Homework (30%): Each week you will be assigned reading and homework to be turned in. The written homework will be evaluated by me and your peers. This homework will generally consist of mathematics problems and/or short writing assignments. You may also have occasional reading quizzes at the discretion of the instructor. Through your reading, homework assignments and class activities you will

- locate, critically read, and evaluate information to solve problems.
- critically evaluate the ideas of others
- apply knowledge and skills within and across the fundamental ways of knowing
- express yourself effectively, professionally, and persuasively both orally and in writing
- develop critical thinking skills
- critically read and analyze sophisticated, complex text
- apply information from within and across various "ways of knowing"

As you critically analyze and evaluate readings from complex text, discuss ideas with me and with your classmates and engage in solving mathematics problems in unique ways, you will develop and hone your critical thinking skills. In addition, many of the readings, as you can see below, will engage you in applying information from within and across various "ways of knowing."

Presentation (35%): During the semester you will be responsible for planning and implementing a 50 minute presentation on a specific topic, teaching that topic to the class, creating homework for the day, designing a grading rubric for the homework, leading the discussion, and evaluating the homework submitted by your classmates. You will utilize sources beyond those required for the class. In addition you will incorporate multiple ways of knowing in your presentation. Two of those ways of knowing will be historical and mathematical. You will incorporate at least one additional way of knowing which (obviously) will be dependent upon your topic. You will meet with me at least two weeks prior to your presentation to discuss plans for your presentation. I will give you feedback and suggestions. Following your presentation, you will then meet with me to discuss your experience. Through this major assignment, you will

- locate, critically read, and evaluate information to solve problems.
- critically evaluate the ideas of others
- apply knowledge and skills within and across the fundamental ways of knowing
- orally express yourself effectively, professionally, and persuasively
- critically read and analyze sophisticated, complex text

- engage in analysis at an advanced level
- develop information literacy skills

This assignment will give you an opportunity to become an expert on a particular topic and will give you valuable experience in planning and carrying out the requirements of a longer presentation. You will also sharpen your information literacy skills as you locate, evaluate, and analyze information to prepare your presentation.

Written Project (35%): You will write a formal research paper on a topic related to the history of mathematics. You will incorporate at least 3 ways of knowing in your paper. Two of them will be historical and mathematical. You will meet with me prior to the 8th week of classes to discuss your paper topic and the scope and expected format. You will incorporate readings and resources that go beyond those used in the class. You will turn in a rough draft of your paper prior to the Thanksgiving break. The final version of your paper is due at the beginning of the scheduled final examination time. Most papers will be at least 20 pages long (double spaced) and will incorporate readings and research beyond those required of each student in the course. I prefer that you use APA format for your research paper, but you may use any other accepted format if you are studying in a field that uses a different citation style. As you complete this assignment, you will

- locate, critically read, and evaluate information to solve problems.
- critically evaluate the ideas of others
- apply knowledge and skills within and across the fundamental ways of knowing
- express yourself effectively, professionally, and persuasively in writing
- critically read and analyze sophisticated, complex text
- engage in analysis at an advanced level
- develop information literacy skills

Class Calendar

Below is a tentative schedule of readings by week and some possible discussion questions for consideration in class. Class discussions will not be limited to these questions and I will often hand out other questions each week as well, so it is important that you thoroughly read and make notes on the readings. The discussions will be based on information in the readings, your prior educational knowledge in mathematics and in the Foundational Studies Program, and your own personal experience. The abbreviations for the required books from which the readings are drawn are given just prior to the schedule.

ME: *The Mathematical Experience* by Davis and Hersch
FLT: *Fermat's Last Theorem: Unlocking the Secret of an Ancient Mathematical Problem* by Aczel
WM: *Women in Mathematics* by Osen
WMA: *Women in Mathematics: The Addition of Difference* by Henrion
EMCII: *The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook*, V.J. Katz, ed.

OHM: *The Oxford Handbook of the History of Mathematics*, Robson and Stedall, eds.
GM: *A Manual of Greek Mathematics* by Heath

Other readings which will be made available.

Note: Each week we will be addressing mathematical and historical ways of knowing. I have indicated other ways of knowing that we may be incorporating each week.

Week 1

Readings:

ME: *Overture, The Mathematical Landscape, Varieties of Mathematical Experience,* pp. 1-55

Possible Discussion Questions:

In "The Ideal Mathematician," can you find any evidence of contradiction between what the ideal mathematician believes and what he can explain to the student? Describe "the Ideal Mathematician." What are some of the issues/problems with this depiction?

Does the development of personal knowledge of mathematics mirror the historical development of the subject? That is, do we learn mathematical concepts as individuals in the same order in which these concepts appeared historically? Give examples to support your stance.

Week 2 (natural sciences)

Readings:

EMCII: pp. 7-51, *Egyptian Mathematics* by Annette Imhausen

OHM: pp. 781-800, *Traditions and myths in the historiography of Egyptian mathematics* by Annette Imhausen.

OHM: pp. 407-428, *Mixing, building, and feeding: Mathematics and technology in ancient Egypt* by Corinna Rossi.

Excerpts from *Mathematics in the Time of the Pharaohs* by Richard J. Gillings
The Recto of the Rhind Mathematical Papyrus, pp. 45-70
The Recto Continued, pp. 71-80

Possible Discussion Questions:

Why are there so few extant samples of Egyptian mathematics? Why is it important to take multiple contextual factors into consideration when studying ancient Egyptian mathematics? Why did early historians of Egyptian mathematics refer to their arithmetical calculations as "primitive"? What biases did they bring to their analyses?

Why do you suppose the author of the Ahmes Papyrus did not choose to say that the double of the thirteenth part is the seventh part plus the ninety-first part, that is, 2/13 = 1/7 +1/91? Why is the relation 2/13 = 1/8 + 1/52 +1/104 made the basis for the tabular entry instead? What does this tell us about how the Egyptians did arithmetic?

Are you convinced by Gilling's analysis and interpretation of the significance of the recto of the Rhind papyrus? Why or why not?

Week 3 (ethics and social responsibility, cultural diversity)

Readings:

EMCII: pp. 57 – 179, *Mesopotamian Mathematics* by Eleanor Robson

OHM: pp. 199-228, *Mathematics Education in an Old Babylonian scribal school* by Eleanor Robson

Possible Discussion Questions:

Describe how the depiction of mathematics education in the Babylonian society is similar to and different from mathematics education in the U.S. today. What was considered appropriate mathematical knowledge for the average person then, and now?

What do the two problems of recovering two numbers from their sum and product or from their difference and product have to do with quadratic equations as we understand them today? Can we conclude that the Babylonians "did algebra"?

What are the advantages of a place-value numeration system? Describe the advantages. Why do you think the Babylonians used a sexagesimal (base 60) number system? What archeological evidence is there for your speculations?

Describe how the creation of large cities necessitated the invention of more complex mathematics than that required of hunter-gatherers or small agrarian communities.

Weeks 4 & 5 (natural sciences, philosophy)

Readings:

GM: all

OHM: pp. 107-132, *The two cultures of mathematics in ancient Greece* by Markus Asper

OHM: pp. 801-826, *Reading ancient Greek mathematics* by Ken Saito

Possible Discussion Questions:

Common sense seems to indicate that a logical development of a theory would have one great advantage over an informal intuitive development, namely that its conclusions would be certain, and one could therefore have much more confidence in them than in the results of vague intuitive arguments. Common sense also seems to indicate that one would pay for this increased certainty by having to give up many appealing intuitive ideas that are too vague to be captured in a logical presentation. Are there any respects in which this common sense is the opposite of the truth? Can you think of any cases where the effect of logical development is to make conclusions seem more, rather than less doubtful and to stimulate the creation of new intuitive possibilities rather than excluding others?

How did Aristarchus and Eratosthenes push the ideas of measuring inaccessible distances first written about by Thales, to understand the natural world? What did they do and how did they do it? How did this lead to future developments in astronomy?

How did the Greek conception of numbers and geometry influence their approach to number theory? How does this approach reflect their view of the world? How does the addition of algebra influence and reflect our view of the world? How did this conception influence (or was influenced by) Greek philosophy?

Although the Greeks defined the conic sections as sections of cones, it is customary in present-day courses in analytic geometry to define them using the focus-directrix definition. What is gained and lost by these two approaches?

Why is Archimedes considered the greatest mathematician of antiquity?

Week 6 (fine and performing arts)

Readings:

EMCII: pp. 515-670, *Mathematics in Medieval Islam* by J. Lennart Berggren

OHM: pp. 301-328, *Patronage of the mathematical sciences in Islamic societies* by Sonja Brentjes

OHM: pp. 827-852, *Number, shape, and the nature of space: Thinking through Islamic art* by Carol Bier

Excerpts from *Islamic Art and Architecture: The System of Geometric Design* by Issam El-Said.

Excerpts from *M.C. Escher: Vision of Symmetry* by Doris Schattschneider

Possible Discussion Questions:

In what ways was the Arab approach to algebra similar to and different from the Greek approach? In what ways did they use geometry?

Describe Al-Khwarizmi's method for solving quadratic equations. How does it connect to "completing the square"? How might you use this idea in a secondary mathematics classroom?

Compare, in its effect on the production and dissemination of knowledge, the Arabic conquest of neighboring lands with the earlier conquests of Alexander the Great and with the conquests of the Romans.

Why is Islamic art and architecture so geometric? How has this art influenced modern artists?

Week 7 (natural sciences)

Readings:

EMCII: pp. 385-510, *Mathematics in India* by Kim Plofker

OHM: pp. 519-536, *Sanskrit mathematical verse* by Kim Plofker

Excerpt from *e: The Story of a Number* by Eli Maor
John Napier, 1614, pp. 3-22

Possible Discussion Questions:

How and why did the invention of logarithms improve the computational abilities of people from mathematicians to scientists to navigators? Why do we still require students to learn logarithms today? What do we primarily use them for now? How does/should this influence how we teach the subject?

Compare and contrast the Greek and the Hindu approaches to mathematics.

Discuss the impact of the base-ten number system on mathematical development throughout India, Asia, and Europe.

How has the study of astronomy furthered the development of mathematics? Consider the Egyptian, Babylonian, Greek, Islamic, and Indian societies.

Week 8 (social and behavioral studies, philosophy)

Readings:

pp. 3-30 in *An Imaginary Tale: The Story of* $\sqrt{-1}$ by Paul J. Nahin

The Geometry of Rene Descartes, Book I, translated from the French and Latin by David Eugene Smith and Marcia L. Latham

Excerpt from in *Mathematics and Civilization* by H.L. Resnikoff and R.O. Wells, Sr.

The Algebrization of Geometry, pp. 202-231

Possible Discussion Questions:

Explain how differences in the way various cultures think about mathematics have mirrored the progression in thinking about pi from a geometric to an analytic to a computational approach.

Discuss the culture of the mathematics community in the time of Cardano and Fontana. How did the lack of collaboration and cooperation among mathematicians of the time hinder the free spread of ideas and the advancement of the subject? What are some other examples (not necessarily from mathematics) of this phenomenon?

How did the marriage of geometry and algebra advance the cause of both? How did it help the progress of our understanding of mathematics and the ability of mathematics to solve practical problems?

What might have been Descartes' reason for introducing his geometric method in the course of explaining his method in philosophy? Why would a book on geometry be relevant to a treatise on philosophy?

Week 9

Readings:

Excerpt from *Mathematical Thought from ancient to Modern Times* by Morris Kline

The Creation of the Calculus, pp. 342-390

Possible Discussion Questions:

Notice that Viete, Fermat, Descartes, and Leibniz were all trained in the law. The law thus seems to have provided the world with a large number of mathematicians. The number of composers who were trained for the law is also impressive, including Handel, Bach, Schumann, and Tchaikovsky. Can you think of anything that mathematics, law, and music have in common that would account for the apparently large number of people who excel in all three?

What is the relationship between Zeno's paradoxes and the calculus? How does the resolution of the paradoxes help with understanding the calculus?

Compare and contrast Newton's and Leibniz's approach to the calculus.

Why is the Calculus considered one of the greatest creations in all mathematics?

Week 10 (ethics and social responsibility, global perspectives and cultural diversity)

Readings:

WM: all

WMA: all

Possible Discussion Questions:

Why is Hypatia the first known woman mathematician of whom we have definitive knowledge?

Compare and contrast the lives of the women mathematicians described in the Osen's book. In what ways do their lives mirror the lives of modern women mathematicians as described by Henrion? How are they different?

Why have there been so few eminent women mathematicians?

Are males better at mathematics than females?

Address Henrion's statements and refutations of myths of women in mathematics. Are you convinced by her arguments? Why or why not? Did any of her arguments or examples resonate with your own experience? How? Do any of these myths apply to other traditionally underrepresented groups? How? As future teachers, what can you do to debunk some of these myths with your future students?

Week 11 (natural sciences, ethics and social responsibility)

Readings:

Excerpts from: *The Lady Tasting Tea: How Statistics Revolutionized Science in the Twentieth Century* by David Salsburg

That Dear Mr. Gosset, pp. 25-32
The Bayesian Heresy, pp. 125-135
Does Smoking Cause Cancer? pp. 181-194

Excerpt from: *The History of Statistics: The Measurement of Uncertainty before 1900* by Stephen M. Stigler

Least Squares and the Combination of Observations, pp. 11-61

Excerpts from: *Statistics on the Table: The History of Statistical Concepts and Methods* by Stephen M. Stigler

Karl Pearson and the Cambridge Economists, pp. 13-50
Regression toward the Mean, pp. 173-188

Foundations of Vital Statistics by John Graunt, in *The World of Mathematics,* James R. Newman, ed., pp. 1416-1435

Mathematics of a Lady Tasting Tea by Sir Ronald A. Fisher, in *The World of Mathematics,* James R. Newman, ed., pp. 1512-1521

Concerning Probability by Pierre Simon de LaPlace, in *The World of Mathematics,* James R. Newman, ed., pp. 1316-1333

Chance by Henri Poincare, in *The World of Mathematics,* James R. Newman, ed., pp. 1374-1394

The Meaning of Probability by Ernest Nagel, in *The World of Mathematics,* James R. Newman, ed., pp. 1395-1414

Possible Discussion Questions:

How is much of modern science founded on statistics? In what ways? How does this affect our lives? In what ways?

What are the origins of modern probability? How did the letters between Fermat and Pascal contribute to this theory? What sorts of problems were they engaged in solving?

In what ways does the general population's ignorance of statistics allow them to be persuaded to adopt particular viewpoints? What does it mean for an electorate to be well-informed? How do we become better informed? What are the political and social ramifications of our work/success/failure as mathematics teachers?

Week 12 (fine and performing arts, philosophy)

Readings:

OHM: pp. 639-662, *Mathematics, music, and experiment in late seventeenth century England* by Benjamin Wardhaugh.

Excerpt from *Mathematical Thought from Ancient to Modern Times* by Morris Kline

Non-Euclidean Geometry, pp. 861-881

Durer as a Mathematician by Erwin Panofsky, in *The World of Mathematics*, James R. Newman, ed., pp. 603-621

Projective Geometry by Morris Kline, in *The World of Mathematics,* James R. Newman, ed., pp. 622-641

Symmetry by Hermann Weyl, in *The World of Mathematics,* James R. Newman, ed., pp. 669-724

Possible Discussion Questions:

Is Euclidean geometry objective truth? Does the axiomatic method establish truth or is it simply a game of deduction? How did the discovery/invention of non-Euclidean geometry change attitudes towards truth and the axiomatic method in the 1800s and beyond?

What are some of the connections between music and mathematics?

Describe how discoveries in mathematics have historically been connected to different trends in art.

Week 13 (global perspectives and cultural diversity)

Readings:

FLT: all

Possible Discussion Questions:

Describe how Andrew Wiles' proof of Fermat's Last Theorem was built on the previous work of other mathematicians.

The proof of Fermat's Last Theorem uses mathematics that is seemingly unrelated to the statement of the Theorem. Give several examples of mathematical topics that at first glance seem unconnected and discuss their connections.

Why was a proof of Fermat's Last Theorem necessary even though most mathematicians believed it was true and it had been verified by computer up to a very large number?

Compare and contrast the collaboration of Wiles and his colleagues with that of Cardano and Fontana when solving the cubic. How has the culture of the mathematics community changed over time? Does this change reflect changes in our own culture? Why or why not and in what ways?

Comment on the following "lessons" of Fermat's Last Theorem: (a) easy statement (understandable by middle school students) but a hard proof (understandable by very few); (b) integration of disparate mathematical ideas; (c) problem chipped away at over centuries; (d) collaborative effort.

Week 14 (global perspectives and cultural diversity, social and behavioral studies, ethics and social responsibility)

Readings:

ME: *Mathematical Models, Utility,* pp. 76-127, *From Certainty to Fallibility,* pp. 317-359, *Mathematical Reality,* pp. 361-411

OHM: pp. 85-104, *The internationalization of mathematics in a world of nations, 1800—1960* by Karen Hunger Parshall

OHM: pp. 755-778, *Abstraction and application: New contexts, new interpretations in twentieth-century mathematics* by Tinne Hoff Kjeldsen

Possible Discussion Questions:

To what extent were developments in Analysis in the 19th century motivated by internal factors in mathematics rather than by the needs and preferences of society? Give specific examples.

Consider the Bernoulli family. Are mathematicians born or made? How do the answers to this question influence teachers' attitudes and beliefs towards mathematical abilities in their students? And, in turn, what are the effects of teachers' attitudes and beliefs on their students?

Week 15 (philosophy, social and behavioral studies)

Readings:

ME: pp. 304-397

OHM: pp. 663-683, *Modernism in mathematics* by Jeremy Gray

Excerpt from *Mathematical Expeditions: Chronicles by the Explorers* by Reinhard Laubenbacher and David Pengelley

Set Theory: Taming the Infinite, pp. 54-94

Goedel's Proof by Ernest Nagel and James R. Newman in *The World of Mathematics*, James R. Newman, ed., pp. 1668-1695

Possible Discussion Questions:

Consider "The Classic Classroom Crisis of Understanding and Pedagogy." As a student of mathematics, where have you experienced such a "crisis" and how did your teacher deal with this? How will you cope with this situation as a teacher? An argument can be made that this happens more often at the secondary level than at the university because at the university the teacher usually possesses more content knowledge. Respond to this argument.

Compare and contrast the impact of Goedel's incompleteness theorem with the impact of discovery of the independence of the parallel postulate, both within and beyond the mathematical world.

How does the formal presentation and products of mathematics differ from its learning and its creation? What are the implications of this on students' perceptions of mathematics?

In what ways do you think that the possession of a conscious (or unconscious) philosophy of mathematics affects the creation of new mathematics? How does it

affect what is valued in mathematics? How does it affect the teaching of mathematics, especially to the average student?

What insights does Georg Cantor's story bring us about the roles of community, individualism, and mental illness in mathematical discovery?

Other Class and University Policies:

Laptop Required for Course: Irregular Usage: For the purposes of this course it will be assumed that you are in compliance with the mandatory laptop policy of the University. You will be expected to bring your laptop and be ready to use it for those class periods noted (below/above). Usage of the laptop must conform to the provisions of this course as laid out in this syllabus as well as the Code of Student Conduct.

Academic Honesty: It is imperative that you adhere to standard practices of academic honesty and integrity in this course. I encourage you to review the University's Academic Dishonesty Policy found in the Student Code of Conduct (see http://www.indstate.edu/sjp/docs/code.pdf). If I find that you have violated any part of the Academic Dishonesty Policy, at minimum, you will receive an F in the course.

Academic Freedom: "Teachers are entitled to freedom in the classroom in discussing their subject, but they should be careful not to introduce into their teaching controversial matter which has no relation to their subject." The preceding comes from the American Association of University Professors statement on academic freedom. Though the entire statement speaks to many issues, it is this portion on conduct of the course that is most relevant. For the purpose of Foundational Studies courses this means that faculty have the right to conduct their class in a fashion they deem appropriate as long as the material presented meets the learning objectives laid out by the entire faculty. http://www.aaup.org/AAUP/pubsres/policydocs/contents/1940statement.htm

American with Disabilities Act Statement: "Indiana State University seeks to provide effective services and accommodation for qualified individuals with documented disabilities. If you need an accommodation because of a documented disability, you are required to register with Disability Support Services at the beginning of the semester. Contact the Director of Student Support Services. The telephone number is 237-2301 and the office is located in Gillum Hall, Room 202A. The Director will ensure that you receive all the additional help that Indiana State offers. If you will require assistance during an emergency evacuation, notify your instructor immediately. Look for evacuation procedures posted in your classrooms."